

# Graphical User Interface (GUI) For Quadratic Assignment Problem (QAP)

**Prof. Ranjit Singh Chauhan**

Director, JMIETI Radaur, Yamuna Nagar, Haryana, India.

<https://orcid.org/my-orcid?orcid=0000-0002-1235-782X>

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**Abstract** The focus of this paper is on Quadratic Assignment Problem (QAP). Assignment problem is a special type of allocation problem. In both cases the objective is to fulfill the targets by means of available resources which are available in specified amounts. MATLAB is high visualization simulation tool for the design and analysis of various engineering problems. The Graphic User Interface (GUI) is designed for this work for good presentation. There are two situations in Assignment Problem i.e. Maximization or Minimization. The data is concerned with Path for minimization and Profit for maximization. The results for the concerned organization are very attractive. In the end, it is verified with an example/case study.

**Keywords:** Assignment, Maximization, QAP, GUI.

## I. INTRODUCTION

During the few decades, the Assignment Problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operations research in mathematics. The classic Quadratic Assignment Problem (QAP) is one of the most interesting and challenging combinatorial optimization problems in existence. Since QAP is NP-complete, it is notoriously difficult to be solved by exact solution methods.

It consists of finding a maximum weight matching in a weighted bipartite graph. The Quadratic Assignment Problem (QAP) was introduced by Koopmans and Beckmann in 1957 as a mathematical model for the location of indivisible economical activities. QAP is often used to describe a location problem.

In its most general form, the problem is as follows: There are a number of agents and a number of tasks. Any agent can be assigned to perform any task, incurring some cost that may vary depending on the agent-task assignment. It is required to perform all tasks by assigning exactly one agent to each task in such a way that the total cost of the assignment is minimized.

If the numbers of agents and tasks are equal and the total cost of the assignment for all tasks is equal to the sum of the costs for each agent (or the sum of the costs for each task, which is the same thing in this case), then the problem is called the Linear assignment problem. Commonly,

when speaking of the Assignment problem without any additional qualification, then the Linear assignment problem is meant.

It was found that a transportation problem is degenerate if, while deriving a feasible solution, an allocation to any cell satisfies the columns as well as row requirements simultaneously. It is observed that in the assignment problem, each resource can be assigned to only one job and each job requires only one resource. Hence the assignment problem is a completely degenerate form of the transportation problem.

The programs/software are Graphical User Interface in nature. MATLAB is high visualization simulation tool for the design and analysis of various problems. The Graphic User Interface (GUI) is designed for good presentation of analysis and synthesis of engineering problems. A GUI offers graphical icons, and visual indicators, as opposed to text-based interfaces, typed command labels or text navigation to fully represent the information and actions available to a user.

## II. QUADRATIC ASSIGNMENT PROBLEM

The assignment problem may be defined as follows: Given  $n$  facilities and  $n$  jobs, and given the effectiveness of each facility for each job, the problem is to assign each facility to one and only one job so that the given measure of effectiveness is optimized.

The assignment problem stated above can be translated into problems in many decision fields. As an example, consider the following situation:

The municipal committee of a city has a fleet of  $n$  tractors located at different places in a city. There are also  $n$  trailers lying at different places in the same city and it is desired to pick up and haul the trailers to the centralized depot. The problem is to assign each of the  $n$  tractors to corresponding trailers in such a way that a given measure of effectiveness (e.g. of total cost involved or the total distance travelled or the total time of travel for tractors) is optimized.

Such a problem can be represented by  $n \times n$  or  $n^2$  matrix which constitutes  $n!$  possible ways of making assignments. One natural method of finding the optimal solution is to enumerate all the  $n!$  possible ways, evaluate their total cost (measure of effectiveness) and select the assignment with minimum cost. It can be easily seen that this basic method becomes extremely laborious even for small or moderate values of  $n$ . For example, when  $n = 10$ , a common situation, the number of possible arrangement is  $n! = 10! = 3,628,800$ . Evaluations of so large a number of arrangements will take a prohibitively large time. This confirms the need of an easy computational technique for solving the assignment problem.

## III. METHODOLOGY

Assignment model may be regarded as a special case of transportation model. Here, the facilities represent the 'sources' while the jobs represent the 'destinations'. The supply available at each source is 1 i.e.,  $a_i = 1$  for all  $i$ . Similarly, the demand at each destination is 1 i.e.,  $b_j = 1$ , for all  $j$ . The cost of 'transporting (assigning) facility  $i$  to job  $j$  is  $C_{ij}$ . The resulting transportation model can

be represented as in the table 1.

		Jobs				Suppl y
		1	2	...	n	
1	1					$a_i$
2	1	$C_{11}$	$C_{12}$	...	$C_{1n}$	1
:	2	$C_{21}$	$C_{22}$	...	$C_{2n}$	1
:	:	:	:	:	:	:
m	1	$C_{m1}$	$C_{m2}$	...	$C_{mn}$	1
D		1	1	...	1	1

**Table 1**  
 Mathematical Representation of Assignment Model

Mathematically, assignment model can be expressed as follows:

Let

$$x_{ij} = \begin{cases} 0, & \text{if the } i\text{th facility is not assigned to } j\text{th job,} \\ 1, & \text{if the } i\text{th facility is assigned to } j\text{th job.} \end{cases}$$

Then, the model is given by

Minimize  $Z =$

$$\sum_{j=1}^n \sum_{i=1}^n C_{ij} \left\{ = \sum_{i=1}^n \sum_{j=1}^n C_{ij} C_{ij} \right\},$$

subject to constraints

$$\sum_{i=1}^n x_{ij} = 1, \quad i = 1, 2, 3, \dots, n, \quad \sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, 3, \dots, n,$$

and  $x_{ij} = 0$  or  $1$ .

If the last condition is replaced by  $x_{ij} \geq 0$ , the transportation model will be there with all requirements and available resources equal to 1.

However, transportation technique cannot be used to solve this model because of degeneracy. Whenever an assignment is made, the row and column requirements will be satisfied simultaneously (rim requirements being equal to 1), resulting in degeneracy. This special structure of assignment model allows a more convenient method of solution.

The technique used for solving assignment model makes use of two theorems.

#### Theorem I

It states that "In an assignment problem, if a constant is added to every element of a row (or column) in the cost matrix, then an assignment which minimizes the total cost on one matrix also minimizes the total cost on the other matrix".

Thus if constants  $u_i$  and  $v_j$  are subtracted from the  $i$ th row and  $j$ th column respectively, then the new cost elements will become.

$$c'_{ij} = c_{ij} - u_i - v_j$$

and the new objective function will be

$$Z' = \sum_i \sum_j C'_{ij} x_{ij} = \sum_i \sum_j (C_{ij} - u_i - v_j) x_{ij}$$

Since, from the constraints of the model

$$\sum_i x_{ij} = \sum_j x_{ij} = 1, Z' = \sum_i \sum_j C_{ij} x_{ij} - \sum_i u_i - \sum_j v_j$$

This shows that the minimization of the new objective function  $Z'$  yields the same solution as the minimization of original objective function  $Z$ .

#### Theorem II

It states "If all  $C_{ij} \geq 0$  and a set  $X_{in} = x_{ij}$  can be such that

$$\sum_i \sum_j C_{ij} x_{ij} = 0,$$

then this solution is optimal".

The above two theorems indicate that if one can create a new  $C'_{ij}$  matrix with zero entries, and if these zero elements, or a subset thereof, constitute a feasible solution, then this feasible solution is the optimal solution.

Thus the method of solution consists of adding and subtracting constants from rows and columns until sufficient number of  $C_{ij}$ 's become zero to yield a solution with a value of zero. The actual procedure for solving assignment models will be described by taking up a few industrial situations. Figure 1 shows the front panel for the problem designed in GUI.



Fig. 1 Front Panel of GUI.

#### IV. SIMULATION RESULTS

The experiment is done on both the maximization and minimization problem. The data is taken for the two problems. The simulation is done for Annexure1 and Annexure II. The Screen shot of GUI for Annexure I is shown in figure 2 below:



Fig.2

The Screen shot of GUI for Annexure II is shown in figure 3 below

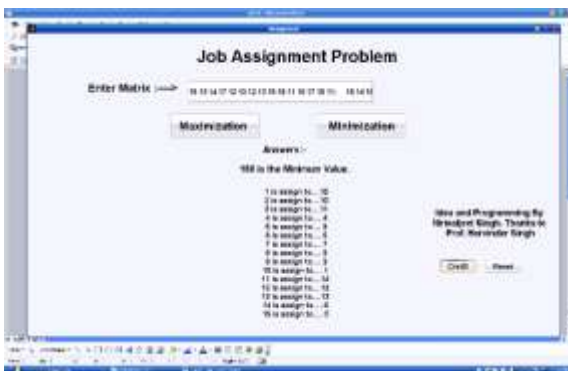


Fig.3

## V. CONCLUSIONS:

In this paper, the GUI is designed for the Quadratic Assignment Problem. There are two situations in Assignment Problem i.e. Maximization or Minimization. The results are very encouraging. The convergence rate of results of the data is very high. It is also concluded that any major problem can also be solved by this work. The paper can be further improved by applying optimization algorithms.

## VI. ANNEXURE 1

A company has 15 trucks in different cities across the region for carrying raw material. The distance between the cities in kilometers is received from the Transportation department of the company (Annexure I).

The work will deal with the assignment of trucks from the different cities so that the total distance covered by the trucks is minimum. The information of distance between various cities in kilometer is shown in the matrix given below: -

```
[15 13 14 17 12 10 12 13 15 18 11 16 17 19 11;  
18 14 16 16 13 11 13 14 17 12 12 17 18 18 13;  
11 16 17 19 21 15 18 15 16 13 10 19 13 15 18;  
15 18 18 11 12 18 14 20 14 16 19 18 19 21 19;  
16 19 19 17 14 16 18 12 11 17 21 11 15 16 17;  
15 17 16 15 11 14 17 19 13 13 20 13 11 17 21;  
13 15 21 18 12 13 10 18 19 14 15 12 14 21 18;  
15 14 18 10 21 15 12 10 18 18 14 14 16 16 14;  
19 12 19 17 20 11 16 16 21 11 12 21 19 21 20;  
11 13 14 16 12 18 11 17 17 19 11 20 18 15 19;  
15 18 14 21 14 12 13 21 18 21 10 12 13 14 20;  
12 17 16 17 13 11 19 13 15 20 12 13 12 16 19;  
17 19 15 20 11 09 17 14 21 19 13 19 10 20 18;  
19 21 14 19 19 10 13 16 20 18 14 14 21 15 17;  
14 20 13 18 14 11 11 15 19 17 19 12 20 17 20]
```

## VII. ANNEXURE 2

Similarly, a company has 10 salesmen and there are 10 cities where the company wants to market its product. The details regarding the profit earned by each salesman in each district is collected from Marketing department (Annexure II).

The work will deal with the assignment of salesmen to various cities which will give the maximum profit.

The information of Profit of each salesman in each city is shown in the matrix given below:-

```
[1040 1050 1065 1080 1090 1015 1030 1020;  
1050 1010 1020 1030 1000 1040 1010 1060;
```

1035 1100 1050 1040 1020 1015 1000 1030;  
1060 1080 1070 2000 1030 1010 1020 1090;  
1050 1040 1030 1020 1010 1000 1070 1060;  
900 1030 1050 1010 1040 1060 1000 1020;  
1110 1070 1020 1060 1000 1050 1040 1030;  
1000 1050 1080 1070 1060 1040 1080 1020]

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